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Influence of Periodic Stiffening on Flanking Noise Transmission—Numerical Analysis Based on Various Frame Designs of a Lightweight Panel Structure

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ABSTRACT

In this paper, a finite-element model is utilized for analysis of noise transmission in a lightweight structure consisting of a source panel subjected to external forces, a receiver panel used to measure the output, and a panel in between acting as a transmission path. Each panel consists of two plates with internal ribs, where it is assumed that the ribs are fully fixed to the plates. A parametric study is carried out on centre panel with regard to various spacing between the ribs. Solid finite elements are adopted for the structure and the computations are carried out in frequency domain in the range below 500 Hz. The responses of the receiving wall are studied under point-force excitation and diffuse-field excitation applied on the source wall. It is found that the positions of the ribs have a significant impact over flanking noise transmission. Thus, a panel with few ribs transmits vibrations in the entire frequency range, whereas a panel with periodic stiffening provides a wide band gap with a significant reduction of the transmitted energy.

Keywords: Lightweight periodic structure, flanking noise transmission, finite-element method.

1. INTRODUCTION

The development in new building structures is requiring a need of lightweight panel structures in dwellings. When designing such new lightweight panel structures, numerical analysis of flanking noise transmission within the structure is a crucial factor to consider.

When examining the structure-borne and airborne flanking noise transmission of a lightweight structure, various methods have been used by researchers [1, 2, 3]. Nightingale [1] found that a full wave statistical-energy analysis (SEA) model of the junction produced useful results regarding the transmission of vibrational energy via flanking junctions from the point of excitation on finite periodic rib-stiffened plates using SEA. However, SEA has limited validity for lightweight structures such as wooden floors with joists spanning in one direction or double-plate panel walls with vertical ribs [2, 4]. So in the present case, the finite-element method (FEM) is considered for numerical analysis of various lightweight panel structures. The FEM can be used [5] to describe flanking transmission in dwellings. Numerical simulations can reduce the cost of experiments and may also improve the design of sound insulation. However, modelling of lightweight structures is complicated, since such structures contain various materials and junctions, and a relatively strong coupling is required between various panels [6]. Further, the FEM has limitations when it comes to the high-frequency range. Small elements must be employed in order to obtain an adequate discretization of the waves propagating in the structure. This results in a huge number of degrees of freedom, leading to long computation times.

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It has been found by experiment as well as computational modelling that variations in the geometry of lightweight structures due to workmanship can influence the acoustic performance of the structure [7]. In this paper, a six-stud double plate structure is analysed with a comparison between a perfectly periodic structure and a structure in which the studs are slightly moved within the frame. The response at the receiving wall is examined under point-force loading on the source wall.

In the present research, flanking noise transmission is the main concern; the acoustic medium in the adjacent room and inside the wall panels is not modelled. Instead, the nodal acceleration levels on the surface of the receiving wall are computed to predict the transmission that can be expected into the room. The lightweight panel structure is tested under concentrated point-force excitation at a nodal point on the exterior surface of the source wall as well as diffuse-field excitation on the wall surface. The present research squarely addresses analysis of flanking noise transmission from the source wall to the receiving wall via the centre wall with various designs of the frame structures. The transmitted kinetic energy and the root-mean-square acceleration levels at the receiving wall are compared. The various designs of the frame between the two plate panels can be described as 6 single-stud ribs placed periodically, three double-stud ribs placed periodically, two double-stud ribs placed periodically, and, finally, a single rib constructed from six studs fixed together. Since the same number of studs is present in all the models, the bending stiffness of the centre wall panel in the height direction is the same.

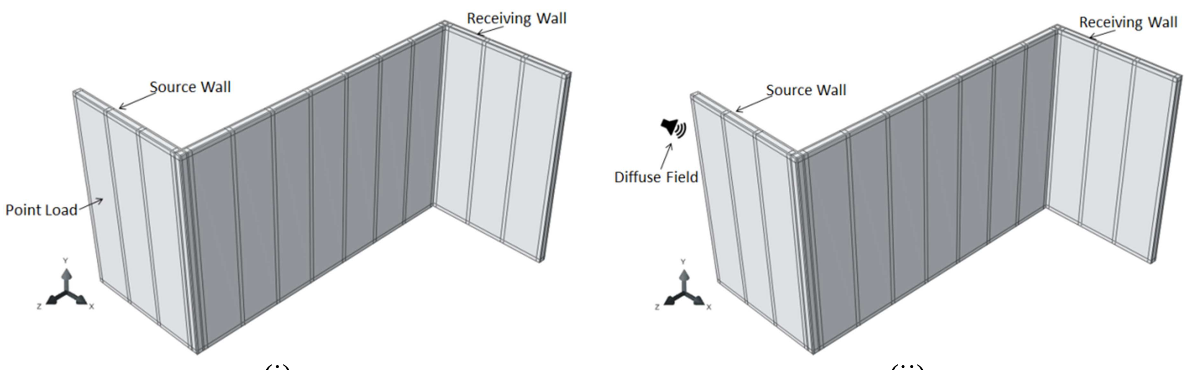
The commercial FEM code ABAQUS has been employed to model the double-plate panel structure using elements available in the ABAQUS/Standard library [8]. In recent work by authors, variation in sound transmission due to the inclusions of acoustic medium and structural damping within double plate panel structure have been examined [9] using a similar model.

The aim of the paper is to get a better evaluation of flanking noise transmission within various frame designs of a lightweight panel structure. Section 2 presents an overview of the computational model of the lightweight panel structure (source wall – centre wall – receiving wall). The results are discussed in Sections 3 and 4, and a summary is provided in Section 4.

2. OVERVIEW OF PROBLEM

Lightweight building structures are usually made by panels with plates on stud or joist frames. To diminish the transmission of sound, frames are usually designed with single or double studs or constructed with layers of foam or another viscoelastic material. In the present case, various frame structures are taken into consideration. The structure consists of a centre wall flanked by two identical walls: a source wall and a receiving wall (cf. Figure 1). The plates are directly attached to the frame.

The aim of the study is to investigate flanking noise transmission via the centre wall under different circumstances. Analyses are carried out on panels with different designs of the frame structure as described in Subsection 2.1. The source wall is subjected to a either a concentrated point force or a diffuse field (see Figure 1) and the kinetic energy transferred to the receiving wall at the other end of the structure is analysed and compared for the various cases. Furthermore, the root-mean-square (RMS) of the acceleration in the normal direction is recorded on one side of the receiving wall to provide an idea about the forces that will be exerted on the air inside the room. However, the acoustic medium within the room has not been included in the present analyses. The model is discussed in Subsection 2.2.



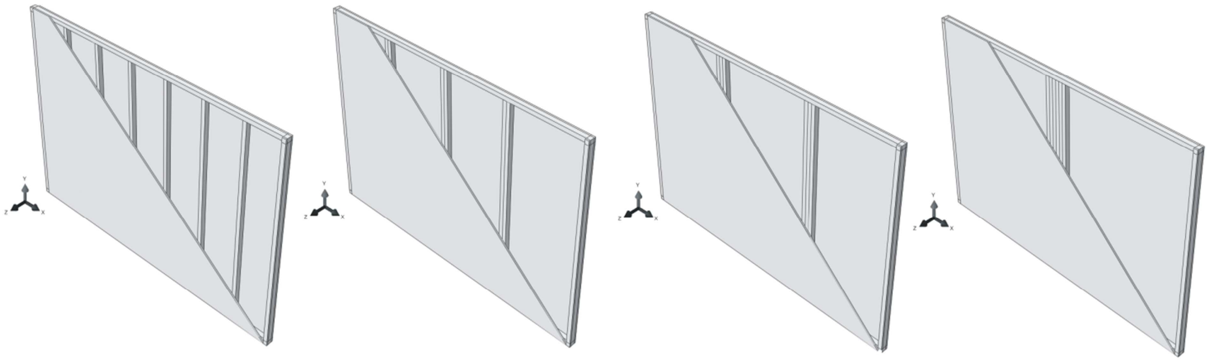
(i) (ii)

Figure 1 – Geometry of panel structure with periodic stiffening by ribs under (i) concentrated point-force excitation and (ii) diffuse-field excitation.

2.1 Panel Descriptions and Materials

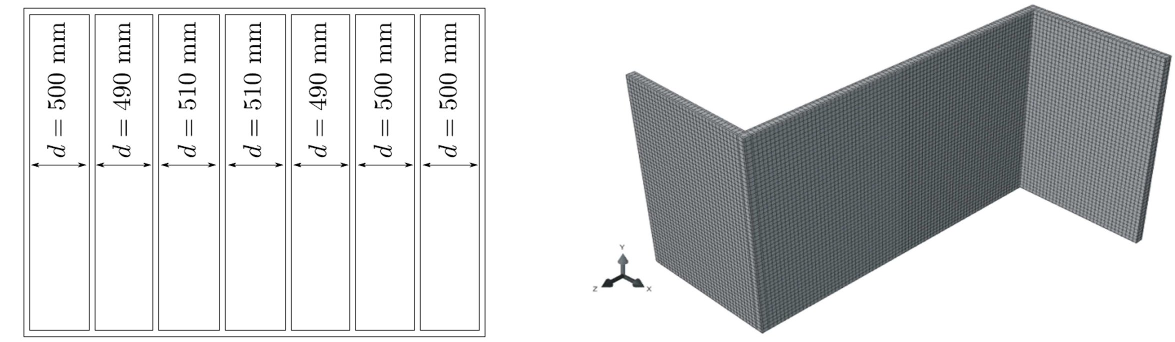
The structure consists of three lightweight panels (source wall – centre wall – receiving wall) made of timber and placed into a z-shape as illustrated in Figure 1. Each panel is constructed as a double plate attached to a timber frame with additional studs acting as internal ribs. The plates have a thickness of 20 mm, whereas the frame has a thickness of 60 mm and each single stud is 50 mm wide. Fixities between adjacent parts of the structure are assumed, i.e. the plates are glued to the frame. Columns with the cross-sectional dimensions 100 mm by 100 mm are put at the corners of the z-shaped panel structure, thus connecting the source wall to the centre wall and the centre wall to the receiving wall.

The source and receiving walls are identical with the dimensions 1675 mm (width) by 2600 mm (height) by 100 mm (thickness). The studs are placed with a distance of 550 mm (centre-to-centre). The centre wall dimensions are 3900 mm (width) by 2600 mm (height) by 100 mm (thickness).



(i) (ii) (iii) (iv)

Figure 2 – Centre panel structure with various frame designs: (i) 6 ribs are placed periodically; (ii) 3 ribs divide the panel into 4 bays; (iii) 2 ribs divide the panel into 3 bays; (iv) 1 rib made from 6 studs divides the panel into 2 bays.



(i) (ii)

Figure 3 – Layout of the computational model: (i) Distance between ribs within the centre panel structure when ribs are moved with slight distance; (ii) finite-element mesh.

A parametric study is carried out with various frame designs for the centre wall. Hence, a total of five different frame structures are taken into consideration to study energy transmission via the centre wall as shown in Figure 2 and Figure 3(i). The five cases can be described as follows:

1. Centre wall with ribs placed with a periodic distance of 550 mm (centre-to-centre) within the frame structure;
2. The configuration of the ribs is similar to Case 1 except that the studs are moved by a small distance within the frame structure as shown in Figure 3(i) to simulate the imperfections in the geometry that may occur due to craftsmanship;
3. Centre wall with a pattern of 3 ribs (the 6 studs are put together two and two to form three ribs dividing the whole panel into four bays);
4. Centre wall with a pattern of 2 ribs (the 6 studs are put together three and three to form two ribs dividing the whole panel into three bays);

5. Centre wall with a pattern of 1 rib (the 6 studs are all fixed together, forming a single wide rib at the centre of the frame and dividing the panel into two bays).

Timber is by nature an isotropic and heterogeneous material. However, in order to simplify the model and keep focus on the effect of the various frame designs, a homogeneous and isotropic material is assumed for the entire structure consisting of the three wall panels. Furthermore, the material is regarded as linear elastic with a Young's modulus of 14 GPa, a Poisson ratio of 0.35and a mass density of 550 kg/m^3. It should be noted that the external air has not been included into the computational model, i.e. the acoustic medium surrounding the walls has been disregarded.

2.2 FE Model

The panel structure is modelled in the commercial finite-element method (FEM) package ABAQUS [8] using solid continuum finite elements. 20-node brick elements with quadratic spatial interpolation of the displacement are adopted with a mesh size of 50 mm. The mesh size has been chosen based on the wavelengths of waves propagating in the model at the higher frequency of interest—in this case 500 Hz. At this frequency, the wavelength of bending waves in the plates is about 100 mm and therefore the accuracy at frequencies near 500 Hz is limited. However, the computational model will still provide some insight into the relative performance of the different centre walls.

The mesh is generated in such a way that nodes constituting the plate mesh align with the nodes on the frame structure. Figure 3(ii) illustrated the mesh on the exterior of the model. All structural contact points are connected using tie constraints in the x, y and z directions. Three-dimensional solid continuum elements have no rotational degrees of freedom, i.e. only displacements are considered. However, due to the local piecewise second-order interpolation of the displacements, the model adequately describes bending in the plates with a single element over the thickness direction.

The panels are fixed along the entire outer edge, i.e. at the top and bottom of the walls as well as the ends of the source and receiver panels that are not connected to the centre panel.

2.3 Excitation

The various designs of the lightweight panel structure are examined under two different loading conditions: (1) Point-force excitation; (2) diffuse-field excitation. In the first case, a concentrated point force is placed on the source wall at x = 250 mm, y = 1300 mm, where the origin of the Cartesian coordinate system is placed at the lower corner of the source panel farthest away from the centre panel. Thus, the concentrated force excites the source panel on the middle of the first bay counted in the direction away from the fixed vertical edge as illustrated in Figure 1(i).

The diffused-field loading condition is generated by a built-in routine in the commercial FEM software ABAQUS. Here it the diffuse field is approximated by a number of deterministic incident plane waves coming from angles distributed over a hemisphere encapsulating the loaded surface. The number of incident plane waves used for the approximation is given by N^2, where N is called the number of seeds. For the present analyses, N = 30 seeds and N = 40 seeds have been employed, so the total numbers of incident plane waves are N^2 = 900 and N^2 = 1600, respectively. The two different numbers of seeds have been included to check the validity of the assumption about a diffuse field, i.e. to examine whether a computational model provides the same results for both seed numbers.

2.4 Method of analysis

Two analyses are performed on the present lightweight structure: 1) Modal analysis; 2) analysis of the steady state response to point force excitation and diffuse field excitation (30 seeds and 40 seeds). In the modal analysis, the real Eigen frequencies and the corresponding Eigen modes are determined for the various frame designs as described in Subsection 2.1. The Lanczos solver implemented in ABAQUS is applied for the structural analysis and all modes occurring below 500 Hz are requested.

In case of the steady state response to point-force excitation and diffuse-field excitation, direct steady state analysis is performed in the frequency domain. Thus, the response is calculated directly from the full stiffness and mass matrices of the global finite-element model rather than a model based on the modal analysis. The steady state response analysis is performed for five different specifications of the model under point-force and diffuse-field excitation:

1. Transmission from source wall to receiving wall when 6 single-stud ribs are placed periodically within the frame of the centre wall (see Figure 2) under point-force and diffuse field excitation;
2. Transmission from source wall to receiving wall when 6 single-stud ribs are placed nearly periodically within the frame of the centre wall (see Figure 3(i)) under point force excitation;

3. Transmission from source wall to receiving wall when 3 double-stud ribs are placed periodically within the frame of the centre panel (see Figure 2) under point-force and diffuse-field excitation;
4. Transmission from source wall to receiving wall when 2 triple-stud ribs are placed periodically within the frame of the centre panel (see Figure 2) under point-force and diffuse-field excitation;
5. Transmission from source wall to receiving wall when 1 rib, 6 studs wide, is placed at the middle of the frame of the centre panel (see Figure 2) under point-force and diffuse-field excitation.

Point-force and diffuse-field excitation are considered in order to quantify the influence of the load position and load type on the transmission of energy to the receiving wall via the centre wall, i.e. the flanking noise transmission. In addition, the structural behaviour of the panel is analysed for two different diffuse fields defined by 30 and 40 seeds, respectively, as explained in Subsection 2.3. Especially, it is identified whether one of the diffuse fields will excite some modes that are not excited by the other diffuse field. It can be expected that small discrepancies arise in the results of the analyses carried out at higher frequencies where the number of seeds is relatively small compared to the number of wavelengths across the source panel.

3. MODAL ANALYSIS

The Eigen modes and the corresponding Eigen frequencies of the panel structure with various frame designs for the centre wall are extracted. Figure 4 shows the accumulated number of modes appearing below a given frequency in the interval from 0 to 500 Hz. It is observed that the number of modes decreases when the studs are divided into more ribs. Hence, the panel with 6 periodic stiffeners has a higher stiffness then all other models, whereas the panel structure with a single wide rib in the centre wall has a comparatively lower stiffness leading to a first structural mode already at 41 Hz.

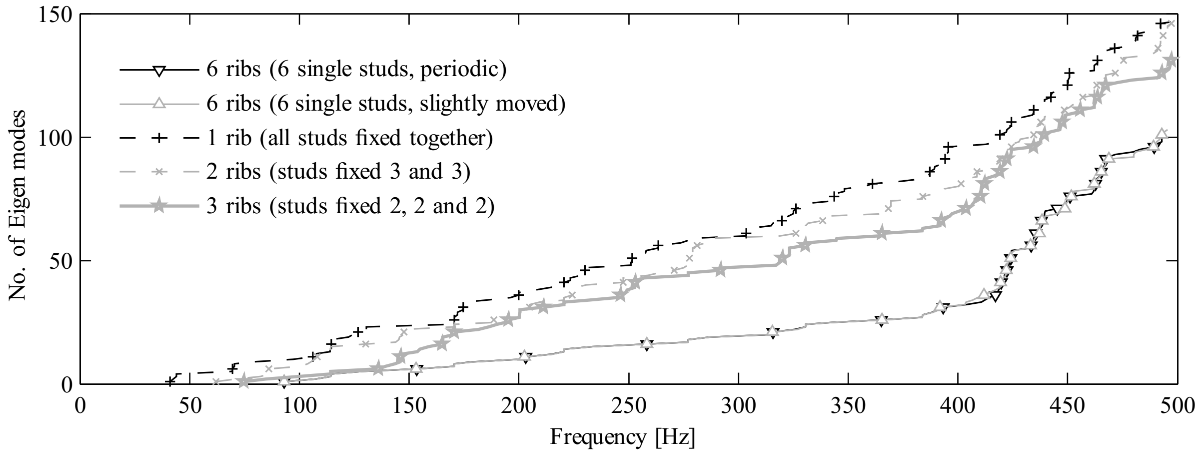
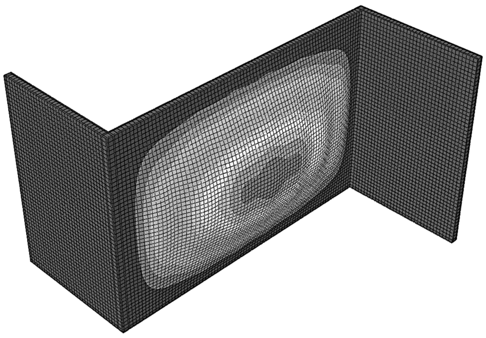


Figure 4 – Eigen frequencies within whole panel structure under various designs of frame structure.

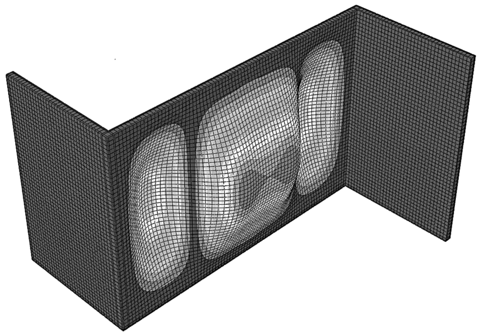
The number of modes is nearly identical when the ribs are placed at strictly periodic distances (Case 1) and when the ribs are slightly moved (Case 2). Thus, the small change in distances between the individual ribs that can be expected due to craftsmanship in real life does not influence the dynamic properties of the building wall panel structure significantly.

The first structural modes in the panels with periodic ribs, a single rib, two ribs and three ribs occur at 93 Hz, 41 Hz, 62 Hz and 75 Hz, respectively. In contrast to the other cases, the first mode shapes in Case 5 (one rib in the centre wall) do not involve bending of the studs, but only bending of the plates. The first mode involving deflection in the wide rib occurs at 79 Hz. The first mode shapes with simultaneous deflection of the plates and ribs for the different cases are shown in Figure 5.

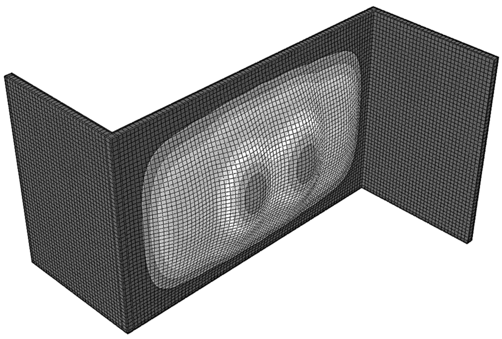
A rapid increase in the number of modes is seen beyond 400 Hz within all panel models. This leads to a higher modal density, especially in Cases 1 and 2 with six single studs placed periodically or nearly periodically in the centre wall. However, still the panel structure with 6 ribs has overall fewer modes and the total number of modes in the analysed frequency range increases with a decrease of the number of ribs. The effect of the higher modal density beyond 400 Hz on the various panel structures under point-force and diffuse-field excitation is described in next section.



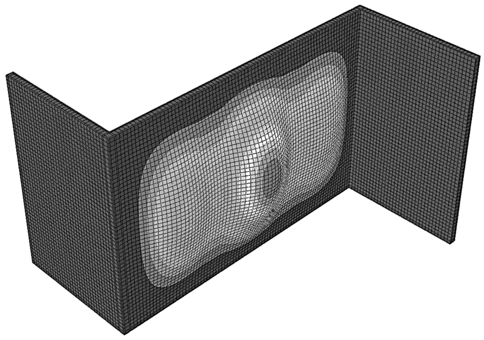
(i) 6 periodic ribs – 93 Hz



(ii) 1 rib (6 studs fixed together) – 79 Hz



(iii) 2 ribs pattern – 62 Hz



(iv) 3 ribs pattern – 75 Hz

Figure 5 – First Eigen mode involving bending of the ribs within the various panel structures: (i) 93 Hz for 6 ribs placed periodically; (ii) 79 Hz for a single wide rib made from six studs; (iii) 62 Hz for 2 ribs, each made by three studs; (iv) 75 Hz for 3 ribs, each made by two studs.

4. PREDICTION OF FLANKING NOISE TRANSMISSION

4.1 Panel-Structure under Point Force Excitation

The steady state response of the structures with various frame designs to point force excitation on the source wall has been analysed. The focus of the analysis has been put on the receiving wall and various designs of the centre wall are considered. The transmission path can be described as source wall – centre wall – receiving wall. The kinetic energy transferred to the receiving wall through centre walls with various frame structures is computed at 55 uniformly distributed frequencies within the frequency range from 60 to 500 Hz. Figure 6 shows the results. Likewise, the root-mean-square (RMS) values of the surface accelerations in the normal direction on one side of the receiving wall are calculated and plotted in Figure 7. The acceleration levels at the surface of the receiving wall are computed to identify whether the acceleration behaviour fully reflects the kinetic energy contained within the receiving wall. It is seen that the kinetic energy and acceleration levels have a similar pattern throughout the entire range of frequencies, since the receiving panel is accelerating at a similar magnitude through the cross section. Due to this, only acceleration levels and not the kinetic energy levels are computed at the frequencies from 60 to 500 Hz in next subsection.

Figures 6 and 7 show that the results are nearly identical for the periodic structure with six ribs made from six single studs and the similar structure in which the ribs are slightly moved. This is explained by the fact that the panels have very similar Eigen modes and frequencies (see Figure 4). The energy transmission levels and acceleration levels are nearly the same in the low-frequency range of 60 to 80 Hz in all of the panel structures, which can be explained by the fact that there are some nearly identical structural modes in the models cf. Figure 8(i). It is seen that there is a sudden decay in the energy levels (see Figure 7) and acceleration levels (see Figure 8) in all of the panel models within the 200 Hz to 340 Hz frequency interval. The phenomenon is more pronounced when more periodic ribs are introduced, and in the case of the centre wall with 6 ribs, the band gap covers the full range from about 180 Hz to about 325 Hz. A reduction of nearly 10 dB is observed in the acceleration response.

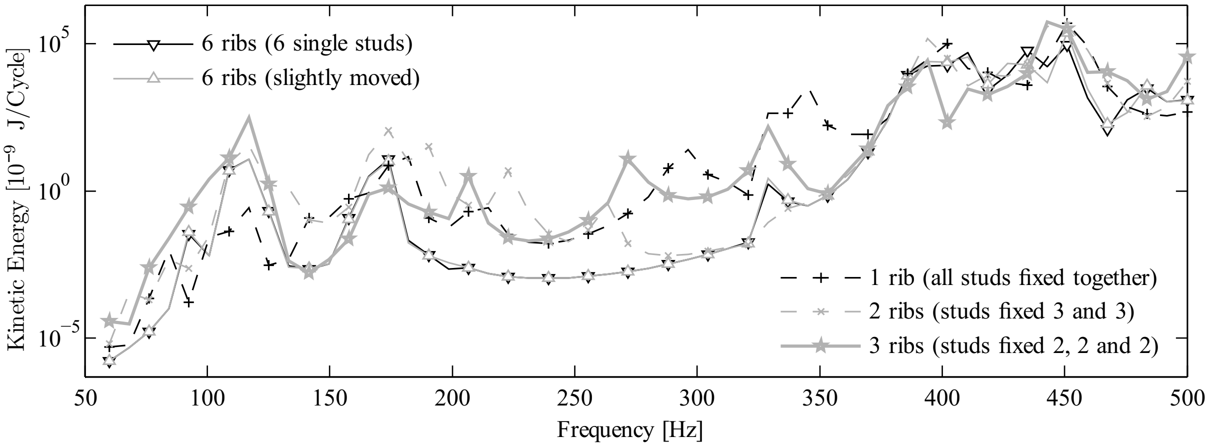


Figure 6 – Kinetic energy levels at various frequencies in the receiving wall under point-force excitation on the source wall.

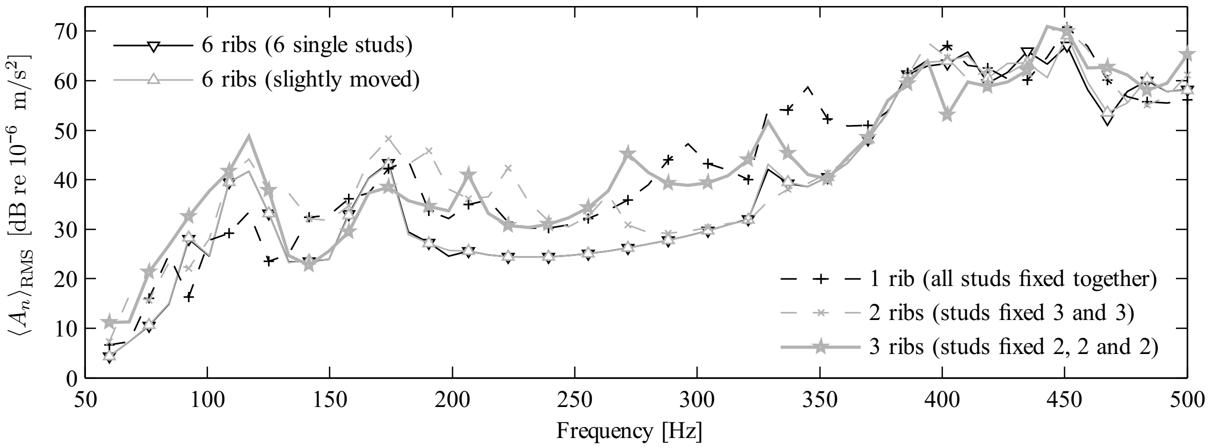
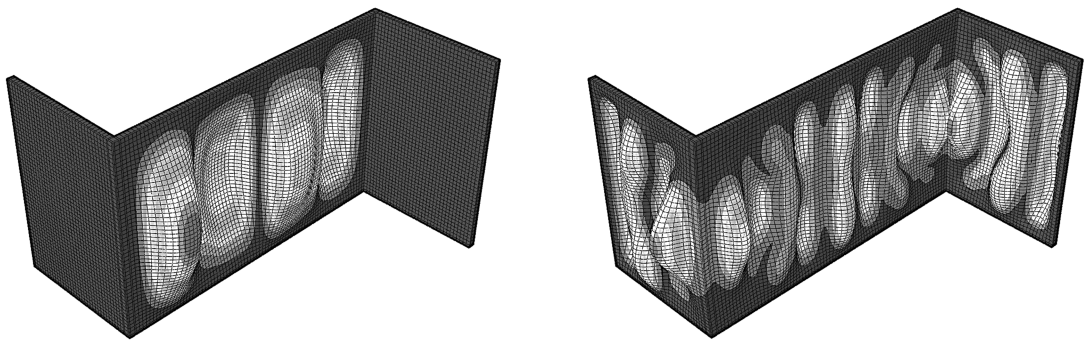


Figure 7 – RMS acceleration levels at various frequencies on the side surface of the receiving wall surface under point-force excitation at source wall surface.



(i) (ii)

Figure 8 – Structural response with nearly the same magnitude in various panel structures: (i) Eigen mode which can be seen at 150 Hz, 97 Hz, 107 Hz and 139 Hz in the periodic 6-ribs, 1-rib, 2-ribs and 3-ribs panel structures, respectively; (ii) Eigen mode which can be observed at around 450 Hz frequency in all of the panel structures.

Overall, the performance of the various panel models under point-force and diffuse-field excitation can be identified in descending order as six single-stud ribs (Cases 1 and 2), three double-stud ribs (Case 3), two triple-stud ribs (Case 4), and one wide rib with all six studs fixed together (Case 5). Only in a narrow frequency interval around 120 Hz, the 1-rib wall performs slightly better, and the centre wall with three double-stud ribs provides less response at 400 Hz. However, the stop band occurring for Cases 1 and 2 within the 180 to 340 Hz frequency range is very significant.

As expected from the high modal densities observed in Figure 4 beyond a frequency of 400 Hz, the transmitted energy and the acceleration levels in the receiving wall behave nearly the same in all of the models at higher frequencies. Here, the various panel structures have structural modes which are quite similar in shape with an example given in Figure 8(ii). These modes are excited in all the structural models, since the force is applied in the same manner and at the same position on the source wall.

4.2 Panel Structure under Diffuse-Field Excitation

In case of diffuse-field excitation, various panel structures are tested under 30 seeds (900 incident plane waves) and 40 seeds (1600 incident plane waves) at 55 uniformly distributed frequencies within the 60 to 500 Hz frequency range. The RMS acceleration is computed for the surface on one side of the receiving wall. At lower frequencies, the acceleration behaviour over the receiving wall surface is seen with a similar pattern for all the panel models due to similar structural modes, see Figures 5 and 8(i).

Compared with the other cases, the transmission of energy is lower in the centre wall panel with six ribs placed periodically under diffuse-field excitation on the source wall. This leads to a lower acceleration of the receiving wall surface, similar to the observation made under point force excitation. Based on the acceleration levels at the receiving wall surface, the performance of the panel structures can be ranked in descending order as periodic centre walls stiffened by six single-stud ribs, three double-stud ribs, two triple-stud ribs and, finally, a single rib consisting of six studs fixed together.

Similarly to the case of point-force excitation, a sudden decay in the acceleration levels is seen between 200 Hz and 340 Hz in—especially in Case 1, i.e. the centre wall with six ribs. Again a wide stop band is clearly identified in this frequency interval. Due to higher modal densities beyond 400 Hz (see Figure 4) within all panel models, it leads to higher acceleration levels in all the panel structures.

It is finally observed that the panel structures tested for 30 seeds and 40 seeds, respectively, are representing nearly the same behaviour along the entire 60 to 500 frequency range, indicating that the fields are indeed diffuse. However, at higher frequencies some discrepancies are observed, which was also to be expected as discussed above. Further, at 270 Hz the models with 30 and 40 seeds behave quite differently. This indicates the existence of a mode in the panel structure that is excited by the diffuse field with 40 seeds but not by the field with 30 seeds.

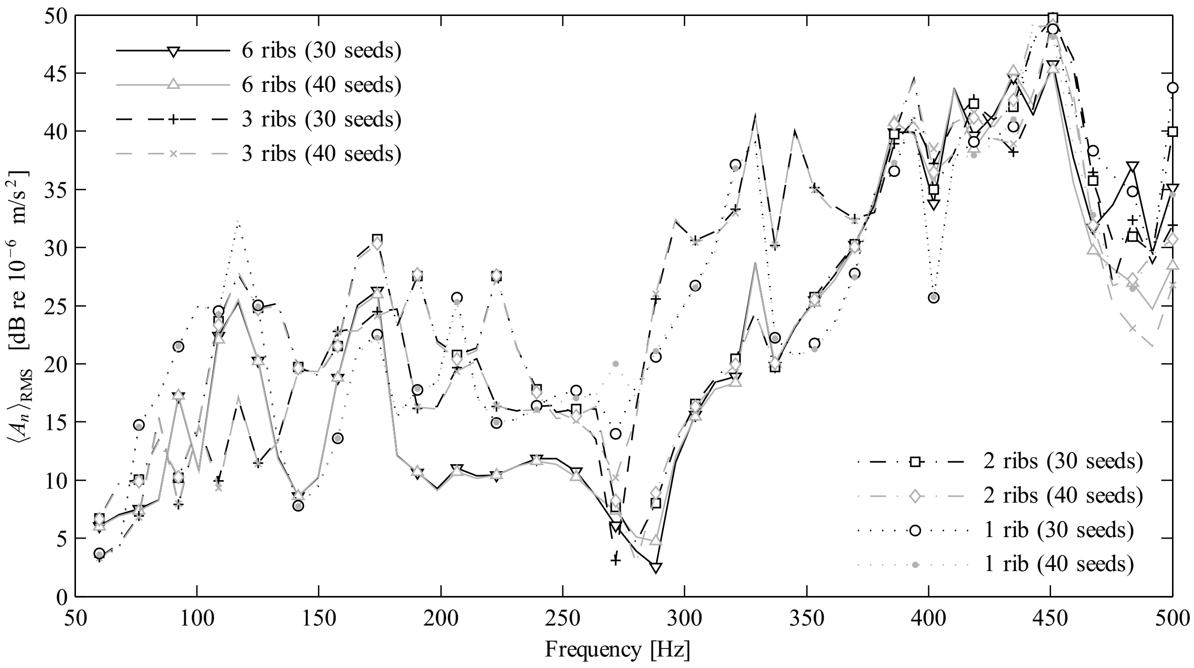


Figure 9 – Acceleration levels at various frequencies on the side surface of the receiving wall under diffuse-field excitation on the source wall surface.

5. Summary

Flanking noise transmission from a source wall to a receiving wall through various centre walls has been analysed for a concentrated point force applied on the source wall and for diffuse-field excitation on the surface of the source wall in the frequency range below 500 Hz. The first structural modes occur at 93 Hz, 92 Hz, 41 Hz, 62 Hz and 75 Hz, respectively, for models in which the centre wall is stiffened by six periodic single-stud ribs, six nearly periodic ribs, a single wide rib (six studs fixed together), two triple-stud ribs, and three double-stud ribs, respectively. This demonstrated that a panel structure with periodic stiffeners has a higher stiffness, even though the same amount of material is used, i.e. the number of studs is the same in all the models.

Under point-force excitation on various panel structures, there are several similarities in the energy propagation in the low-frequency range from 60 to 80 Hz due to the occurrence of similar structural modes in the various models. Such similarities of the response are also seen for frequencies beyond 400 Hz due to a high modal density. An interesting comparison is done with regard to the computation of kinetic energy in the receiving wall and the RMS acceleration of the receiving wall surface, which shows that the two measures of the responses are fairly similar for all of the panel structures.

In case of diffuse-field excitation, the various panel structures are tested with two different models of the diffuse field obtained by combination of 900 and 1600 incident plane waves, respectively. A similar response is observed in the entire range of frequencies as was found in the case of point-force excitation. Stop bands are produced in the 200 to 325 Hz frequency range in all of the panel structures under point-force and diffuse-field excitation, which leads to a sudden decay in energy transmission in this frequency interval. However, the band gab is wider and the reduction in transmission is much greater when the number of periodic stiffeners is increase. Thus, the model with a single, wide rib in the middle of the centre wall provides a poor reduction of the energy transmission compared the centre wall with six single-stud ribs. The structural response of the panel structure under diffuse-field excitation based on 900 and 1600 plane incident waves are observed to be nearly identical below 500 Hz with small discrepancies occurring towards the higher end of the frequency range. It is found that the overall vibro-acoustic behaviour of a lightweight panel structure can be significantly changed by altering the positions of the ribs. However, a significant change of the rib positions is required

Future work involves a closer investigation of the influence of periodicity in the stiffening of lightweight structures by means of examining the occurrence of stop bands and energy dissipation at various junctions. The acoustic medium will be introduced in the adjacent rooms in order to predict flanking noise behaviour of the structure directly. The aim is to mitigate flanking noise transmission via joints as well as direct transmission between adjacent rooms by design of the periodic stiffening.

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